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**A COMPARISON OF THREE MODELS TO PREDICT LIQUIDITY
FLOWS BETWEEN BANKS BASED ON DAILY PAYMENTS
TRANSACTIONS**

By

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A Comparison of Three Models to Predict Liquidity Flows Between Banks Based on Daily Payments Transactions

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Abstract. The analysis of payment data has become an important task for operators and overseers of financial market infrastructures. Payment data provide an accurate description of how banks manage their liquidity over time. In this paper we compare three models to predict future liquidity flows from payment data: 1) a moving average model, 2) a linear dynamic system that links the inflow of banks with their outflow, and 3) a similar dynamic system but with a constraint that guarantees the conservation of liquidity. The error graphs of one-step-ahead predictions on real-world payment data reveal that the moving average model performs best, followed by the dynamic system with constraint, and finally the dynamic system without constraint.

Keywords: Large-Value Payment Systems, Predictive Modeling, Dynamic System, Time-series Analysis

JEL classification: C32, C53, C61, E42, E44, E47

1 Introduction

The global financial crises demonstrated that liquidity problems at banks can occur suddenly, without (long-term) warnings, and cause a devastating impact on a financial system as a whole. Banks execute many payments in large value payment systems to facilitate their own business and that of their customers¹. When these financial processes are somehow disrupted, liquidity problems at banks can occur quickly and propagate across banks through the interconnected structure of a financial system.

¹ The large value payment system of the Eurosystem, currently TARGET2, settles approximately 350,000 payment transactions on a daily basis which corresponding to a total value of 2,300 billion euros.

Payment data are considered a valuable source of information to spot signs of liquidity problems. Payment data are data generated by the processing of payments in large-value payment systems. They include details about the sending and receiving bank, the amount of liquidity that is transmitted, the settlement date, and the payment type. Large sets of historic payment data provide a blueprint of the liquidity flows between banks and allow to be analyzed to identify irregularities.

Indeed, descriptive statistics of large sets of payment data reveal that banks with liquidity problems behave noticeably differently. Among other findings, (Heijmans and Heuver, 2014) show that they tend to pay higher interest rates for interbank loans, use more collateral for inter-day credit, and delay their payments to the end of the day. Therefore, it is of great value for operators and overseers of financial market infrastructures to understand liquidity flows between banks, and know how they are likely going to change in the near future. In this way, potential liquidity problems can be identified at an early stage.

In this paper we compare three models to predict liquidity flows between banks from payment data: a moving average model, and two variations of a linear dynamic system. The main idea behind the models is to aggregate payments over time intervals and use these aggregated liquidity flows to predict future liquidity flows. However, the models do this in slightly different ways. We discuss these differences and show how they can be constructed from historic payment data. Furthermore, we evaluate the predictions made by the models on payment data extracted from the TARGET2 payment system.

The remainder of this paper is organized as follows. Section 2 provides a brief overview of similar models in the literature. Section 3 defines the moving average model and the linear dynamic system. Section 4 describes the evaluation of the models and discusses the results. Finally, section 5 concludes the paper and provides directions for future research.

2 Related Research

There are many ways in which liquidity flows between banks have been modeled. Studies on contagion in interbank markets commonly use the model introduced by (Upper and Worms, 2004). In this model, liquidity flows are defined by a square matrix, say \mathbf{X} , where x_{ij} denotes the total

liquidity that bank i sends to bank j . \mathbf{X} is usually populated from balance sheet data or payment data. Once the elements of the matrix are known, one or more rows are set to zero to mimic the default of the corresponding banks. Several algorithms have been developed that can calculate the effect of the defaults on the remaining liquidity flows in the model.

Simulation studies commonly use an agent-based model to simulate liquidity flows between banks in a payment system. An agent-based model consists of a set of banks (agents) whose behavior is entirely defined by a set of decision rules. These rules specify when banks initiate payments, how much liquidity they transmit, or how they react to changes in the payment system. For example, in (Galbiati and Soramäki, 2011), banks initiated payments to other random banks at timings following a Poisson distribution each simulated day. By changing the rules, e.g. adding payment delays, the effect on the liquidity flows can be estimated.

Some simulation studies also use historic payment data and sequentially process a series of payments in a simulator. A simulator that uses this approach is the BoF-PSS2 simulator. It implements the features of different types of payment systems and allows to 're-settle' payments under different circumstances (Leinonen and Soramäki, 1999).

The models discussed in this paper apply a similar approach as the one used in the model of (Upper and Worms, 2004). Liquidity flows between banks are defined by matrices, as they provide a convenient representation for analysis. However, instead of using one matrix to define liquidity flows at a particular point in time, we use a set of matrices that each model liquidity flows at a particular time interval.

3 Models

Let $B = \{b_1, \dots, b_n\}$ be a set of banks participating in a large-value payment system. The banks transmit liquidity to each other on a real-time and gross basis. This means that, when a bank initiates a transaction, it is directly settled by the payment system on a one-to-one basis. Furthermore, let $T = \langle t_1, \dots, t_k \rangle$ be an ordered set of time intervals, where $t_1 = [\tau_0, \tau_1)$, $t_2 = [\tau_1, \tau_2)$, and so on. We assume that the time intervals in T are consecutive and of equal duration. T may, for example, denote the consecutive openings days of a payment system for a given period of time. For convenience, we define $t \in T$ as an arbitrary time interval,

$t + 1 \in T$ as the time interval that follows directly after t , $t + 2 \in T$ as the time interval after $t + 1$, and so on.

The liquidity that banks transmit to each other is recorded. Let \mathbf{A}^t denote the n by n matrix:

$$\mathbf{A}^t = \begin{bmatrix} a_{11}^t & \cdots & a_{1n}^t \\ \vdots & \ddots & \vdots \\ a_{n1}^t & \cdots & a_{nn}^t \end{bmatrix} \quad (1)$$

where, $a_{ij}^t \in [0, \infty)$ denotes the sum of all liquidity that b_i sends to b_j in time interval t . Furthermore, let $\mathbf{a}_{i\leftarrow}^t$ and $\mathbf{a}_{i\rightarrow}^t$ be the column vectors:

$$\mathbf{a}_{i\leftarrow}^t = \begin{bmatrix} a_{1i}^t \\ \vdots \\ a_{ni}^t \end{bmatrix} \quad \text{and} \quad \mathbf{a}_{i\rightarrow}^t = \begin{bmatrix} a_{i1}^t \\ \vdots \\ a_{in}^t \end{bmatrix} \quad (2)$$

Here, $\mathbf{a}_{i\leftarrow}^t$ denotes all liquidity that b_i receives in time interval t and $\mathbf{a}_{i\rightarrow}^t$ denotes all liquidity that b_i sends in time interval t . We will refer to these vectors as the inflow and outflow of b_i respectively. It is important to note that a_{ii}^t is both an element of the inflow as well as the outflow. It denotes the accumulated savings of b_i up to time interval t in the dynamic system, as will be explained in section 3.2.1. In the moving average model, this liquidity flow denotes the liquidity transmitted by b_i at interval t between subsidiary accounts.

3.1 Moving Average Model

We first define the moving average model. We construct a moving average model for each bank b_i that predicts $\mathbf{a}_{i\rightarrow}^{t+1}$ for the next time interval. It is defined as:

$$\hat{\mathbf{a}}_{i\rightarrow}^{t+1} = \frac{1}{z} \sum_{j=0}^{w-1} \theta_j^i \mathbf{a}_{i\rightarrow}^{t-j} \quad (3)$$

where, w denotes the window size, θ_j^i is the weight corresponding to the outflow of b_i at time interval $t - j$, and z is:

$$z = \sum_{j=0}^{w-1} \theta_j^i \quad (4)$$

In our case, the weights are set to $\theta_j^i = 1$. This implies that $\hat{\mathbf{a}}_{i \rightarrow}^{t+1}$ is the mean of the outflow of b_i over w previous time intervals. Such model is also known as a simple moving average model.

3.2 Linear Dynamic System

Next, we define the linear dynamic system. It is based on the observation that a payment system is essentially a closed system in which liquidity is conserved over time. We aim to exploit this property by predicting the outflow of banks at the next time interval based on their current inflow while making sure that liquidity is conserved.

3.2.1 Conservation of Liquidity

An important constraint is that banks cannot transmit more liquidity than they have available at any moment in time. We call this constraint the conservation of liquidity. It is implemented in our system by requiring the total inflow of a bank at time interval t to equal its total outflow at time interval $t + 1$:

$$\text{inflow}(t) = \sum_{l=1}^n a_{li}^t = \sum_{m=1}^n a_{im}^{t+1} = \text{outflow}(t + 1) \quad (5)$$

During the transition from time interval t to $t + 1$, banks transmit their liquidity from the inflow to the outflow. However, a bank can keep its accumulated savings by sending liquidity to itself. We denote this liquidity flow by a_{ii}^{t+1} . It follows from equation 5 that a_{ii}^{t+1} is the difference between the inflow of b_i at time interval t and the outflow to banks other than itself at time interval $t + 1$:

$$a_{ii}^{t+1} = a_{ii}^t + \sum_{l \neq i} a_{li}^t - \sum_{m \neq i} a_{im}^{t+1} \quad (6)$$

Futhermore, it follows from equation 5 that the payment system is closed, i.e.:

$$\sum_{l=1}^n \sum_{m=1}^n a_{lm}^t = C \quad (7)$$

where, C denotes the total amount of liquidity that is available to all banks in the payment system combined, and is independent of t . Banks cannot create nor destroy liquidity. Instead, they can only meet their liquidity requirements by trading liquidity with others. This is a reasonable assumption from a practical point of view. In many payment systems, only central banks are able to create or destroy liquidity, which they normally do sporadically and to a limited extent.

3.2.2 Regression Model of a Single Bank

We construct a linear regression model for each bank b_i that independently predicts $\hat{\mathbf{a}}_{i \rightarrow}^{t+1}$ for the next time interval given $\mathbf{a}_{i \leftarrow}^t$ at the current time interval. It is defined as:

$$\begin{bmatrix} \hat{a}_{i1}^{t+1} \\ \vdots \\ \hat{a}_{in}^{t+1} \end{bmatrix} = \begin{bmatrix} \theta_{11}^i & \cdots & \theta_{1n}^i \\ \vdots & \ddots & \vdots \\ \theta_{n1}^i & \cdots & \theta_{nn}^i \end{bmatrix} \begin{bmatrix} a_{1i}^t \\ \vdots \\ a_{ni}^t \end{bmatrix} + \begin{bmatrix} \epsilon_{1i}^t \\ \vdots \\ \epsilon_{ni}^t \end{bmatrix} \quad (8)$$

or in matrix notation:

$$\hat{\mathbf{a}}_{i \rightarrow}^{t+1} = \mathbf{\Theta}^i \mathbf{a}_{i \leftarrow}^t + \boldsymbol{\epsilon}_{i \leftarrow}^t \quad (9)$$

where, $\mathbf{\Theta}^i$ is a n by n matrix of model parameters, and $\boldsymbol{\epsilon}_{i \leftarrow}^t$ is a column vector of n error terms. Each parameter $\theta_{jl}^i \in [0, \infty)$ denotes the ratio of liquidity that b_i receives from b_l in time interval t and sends to b_j in time interval $t + 1$. Deviations between the predicted outflow and the actual outflow are denoted by the error terms in $\boldsymbol{\epsilon}_{i \leftarrow}^t$. We assume that $\boldsymbol{\epsilon}_{i \leftarrow}^t \sim \mathcal{N}(0, \Sigma)$ is normally distributed with zero mean and covariance matrix Σ .

A requirement of the model in equation 9 is that it conserves liquidity over time as defined by equation 5. We will show that equation 5 can be implemented by requiring the columns of $\mathbf{\Theta}^i$ to add up to one. By doing so, $\mathbf{\Theta}^i$ becomes a stochastic matrix.

Theorem 1. *The total expected outflow of b_i at time interval $t + 1$ is equal to the total inflow of b_i at time interval t if and only if all columns of Θ^i add up to one:*

$$\sum_{m=1}^n \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{l=1}^n a_{li}^t \quad \text{iff} \quad \sum_{j=1}^n \theta_{jl}^i = 1 \quad \text{for } l = 1, \dots, n \quad (10)$$

Proof. Let b_i be an arbitrary bank and t be an arbitrary time interval. From equation 9 it follows that:

$$\sum_{m=1}^n \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{j=1}^n \sum_{l=1}^n \mathbb{E}(\theta_{jl}^i a_{li}^t + \epsilon_{ji}^t) \quad (11)$$

$$= \sum_{j=1}^n \sum_{l=1}^n \theta_{jl}^i a_{li}^t \quad (12)$$

Let x_l^i be the sum of column l in Θ^i :

$$x_l^i = \sum_{j=1}^n \theta_{jl}^i \quad (13)$$

Then:

$$\sum_{m=1}^n \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{l=1}^n x_l^i a_{li}^t \quad (14)$$

On one hand, if $x_l^i = 1$, then:

$$\sum_{m=1}^n \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{l=1}^n a_{li}^t \quad (15)$$

equals the total inflow of bank b_i at time interval t . On the other hand, if:

$$\sum_{m=1}^n \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{l=1}^n x_l^i a_{li}^t = \sum_{l=1}^n a_{li}^t \quad (16)$$

holds for all a_{li}^t , then $x_l^i = 1$ for all l . \square

3.2.3 Aggregated Dynamic System

By merging the regression models of the banks we obtain a linear dynamic system that maps \mathbf{A}^t to \mathbf{A}^{t+1} . Based on equation 9 as the model definition of a single bank, we can define this system as:

$$\text{vec}((\hat{\mathbf{A}}^{t+1})^T) = \mathbf{D} \cdot \text{vec}(\mathbf{A}^t) + \text{vec}(\mathcal{E}^t) \quad (17)$$

Here, $\text{vec}(\mathbf{X})$ is the vector of all columns of some n by n matrix \mathbf{X} vertically enumerated:

$$\text{vec}(\mathbf{X}) = [x_{11}, \dots, x_{n1}, \dots, x_{1n}, \dots, x_{nn}]^T \quad (18)$$

$\mathbf{D} = \text{diag}(\boldsymbol{\Theta}^1, \dots, \boldsymbol{\Theta}^n)$ is a n^2 by n^2 block matrix, and $\mathcal{E}^t = [\epsilon_{1\leftarrow}^t, \dots, \epsilon_{n\leftarrow}^t]$ is a n by n matrix of errors terms. Clearly:

$$\text{vec}((\hat{\mathbf{A}}^{t+1})^T) = \mathbf{P} \cdot \text{vec}(\hat{\mathbf{A}}^{t+1}) \quad (19)$$

where, \mathbf{P} is a n^2 by n^2 permutation matrix. Therefore, we can rewrite equation 17 as:

$$\text{vec}(\hat{\mathbf{A}}^{t+1}) = \mathbf{P}^{-1} \mathbf{D} \cdot \text{vec}(\mathbf{A}^t) + \mathbf{P}^{-1} \cdot \text{vec}(\mathcal{E}^t) \quad (20)$$

Because $(\mathbf{A}^T)^T = \mathbf{A}$, \mathbf{P} is equal to its own inverse $\mathbf{P} = \mathbf{P}^{-1}$. So, equation 20 is equal to:

$$\text{vec}(\hat{\mathbf{A}}^{t+1}) = \mathbf{PD} \cdot \text{vec}(\mathbf{A}^t) + \mathbf{P} \cdot \text{vec}(\mathcal{E}^t) \quad (21)$$

Now, by defining $\mathbf{y}^t = \text{vec}(\mathbf{A}^t)$, $\mathbf{M} = \mathbf{PD}$, and taking expectations we can rewrite equation 21 as:

$$\mathbb{E}(\hat{\mathbf{y}}^{t+1}) = \mathbf{M} \cdot \mathbf{y}^t \quad (22)$$

3.2.4 Estimation of the Parameters

The elements of the $\boldsymbol{\Theta}^i$ matrices in \mathbf{M} can be estimated from historic payment data. Because the regression models of the banks are independently constructed, we can estimate each $\boldsymbol{\Theta}^i$ separately. We do this by least squares estimation with constraints, guaranteeing the conservation of liquidity (equation 5) and the non-negativity of the elements. The sum of squared errors over k time intervals is:

$$f(\hat{\Theta}^i) = \sum_{t=1}^k \|\epsilon_{i\leftarrow}^t\|^2 \quad (23)$$

$$= \sum_{t=1}^{k-1} \|\hat{\mathbf{a}}_{i\rightarrow}^{t+1} - \hat{\Theta}^i \mathbf{a}_{i\leftarrow}^t\|^2 \quad (24)$$

Taking in account the constraints, this leads to the following optimization problem:

$$\begin{aligned} & \underset{\hat{\Theta}^i}{\text{minimize}} && f(\hat{\Theta}^i) \\ & \text{subject to} && \hat{\theta}_{jl}^i \geq 0 \quad \text{for } j, l = 1, \dots, n \\ & \text{and} && x_l^i = \sum_{j=1}^n \hat{\theta}_{jl}^i = 1 \quad \text{for } l = 1, \dots, n \end{aligned} \quad (25)$$

To solve the optimization problem in 25, we start with a $\hat{\Theta}^i$ matrix having each $\hat{\theta}_{lm}^i = 1/n$. Then, we apply an iterative optimization algorithm to minimize the sum of squared errors. In this paper we applied Augmented Lagrangian (Conn et al., 1991) with Limited-Memory BFGS (Liu and Nocedal, 1989) as implemented in the NLOpt² software package.

3.2.5 One-step-ahead Prediction

The dynamic system can be applied to predict liquidity flows by one-step-ahead prediction with a sliding window. Let w be the window size. First, we use a window consisting of the actual liquidity flows from time interval $t - w$ to t to estimate the $\hat{\Theta}^i$ matrices of $\hat{\mathbf{M}}$ in equation 22. Then, we use the system to predict the liquidity flows at time interval $t + 1$. These steps can be repeated by moving the sliding window forward by one day and predicting time interval $t + 2$, and so on.

4 Evaluation

We evaluated the prediction error of the moving average model and the dynamic system on real-world data. Besides these models, we also evaluated an 'unconstrained' variant of the dynamic system. The models are

² Steven G. Johnson, The NLOpt nonlinear-optimization package. See for more details: <http://ab-initio.mit.edu/nlopt>

denoted by MA , DS_c , and DS_u respectively. DS_u is similar to DS_c except that it does not satisfy the conservation of liquidity property. That is, the elements of the $\hat{\Theta}^i$ matrices of $\hat{\mathbf{M}}$ of DS_u in equation 22 are non-negative but their columns do not add up to one.

4.1 Payment Data

The models were evaluated on a data set of inter-bank payments extracted from the Dutch part of the TARGET2 payment system. The data set consisted of 187,697 payments that were settled in the period of March to April 2015 between 40 banks. These banks were selected as they transmitted the most liquidity during this period. We aggregated the payments over 42 consecutive operational days and calculated the liquidity that the banks transmitted to each other at each day.

The total liquidity available to the banks in the data fluctuated over time as shown by Fig 1. The reason for these fluctuations was that we only modeled inter-bank payments and between a subset of banks in the payment system. Banks were able to make other type of payments and to banks that were outside the scope of our data. As a result, liquidity was not conserved in the data. Moreover, the accumulated savings of the banks in the dynamic systems could not be exactly calculated. Instead, they only consisted of the liquidity transmitted between subsidiary accounts.

4.2 Error Function

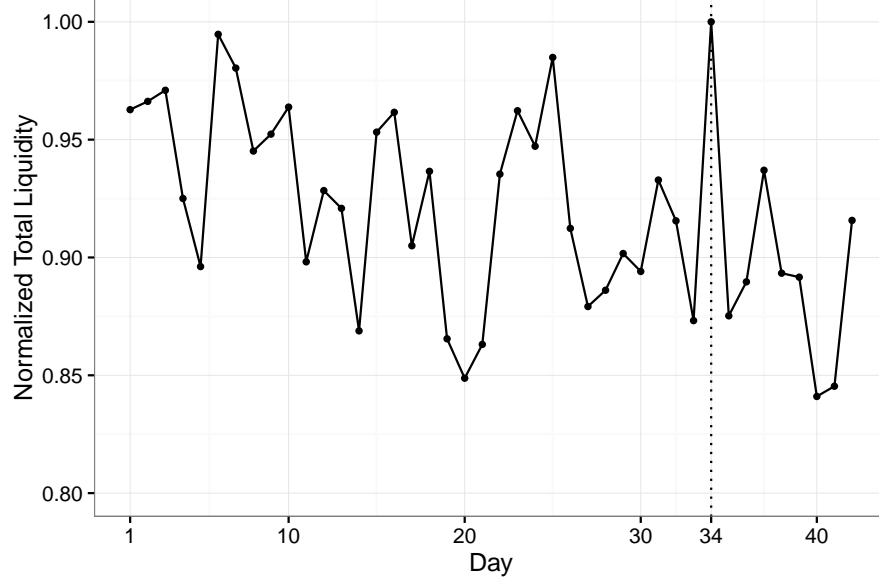
We used the models to predict the liquidity flows of the banks in the data and estimated their prediction error. The prediction error for a single day was measured by:

$$E(t) = \frac{\sum_{l=1}^n \sum_{m=1}^n |\hat{a}_{lm}^t - a_{lm}^t|}{\sum_{l=1}^n \sum_{m=1}^n a_{lm}^t} \quad (26)$$

Hence, $E(t)$ is the proportion of the total incorrectly predicted liquidity to the total actual liquidity at day t . Furthermore, we calculated the average error over the sequence of predicted days when using a sliding window of size w :

$$AE(w) = \frac{1}{p} \sum_{i=1}^p E(t_{w+i}) \quad \text{for } w < k \quad (27)$$

Fig. 1. The total liquidity that was transmitted between the banks at each day in the data set. Liquidity is normalized by dividing it by the maximum total liquidity (occurring at day 34) that was transmitted at a single day for the entire period.



where, $p = k - w$ denotes the number of predicted days. We experimented with different values of w to determine the effect of the window size on the prediction error.

4.3 Results

Table 1 shows the average error of the models for predictions made using window sizes of 15, 20 and 25 days. MA predicted the liquidity flows the best with an average error of about 28%, followed by DS_c achieving an average error of about 35%, and finally DS_u having an average error of about 45%. The window size had only a minimal effect on the predictions of the models. A larger window slightly decreased the average error.

The daily predictions errors of the models for the different window sizes are depicted in Fig 2, 3 and 4. The error curves of MA and DS_c were quite correlated. In contrast, the error curves of DS_u displayed some outliers which considerably affected its average error. The outliers represent cases where the model was over-fitting.

Table 1. Average error of the one-step-ahead predictions made using window sizes of 15, 20 and 25 days.

	$AE(15)$	$AE(20)$	$AE(25)$
MA	0.2819	0.2817	0.2768
DS_c	0.3591	0.3568	0.3473
DS_u	0.4472	0.4498	0.4342

Fig. 2. Daily error curves (measured in percentages) for the one-step-ahead predictions with a sliding window of 15 days.

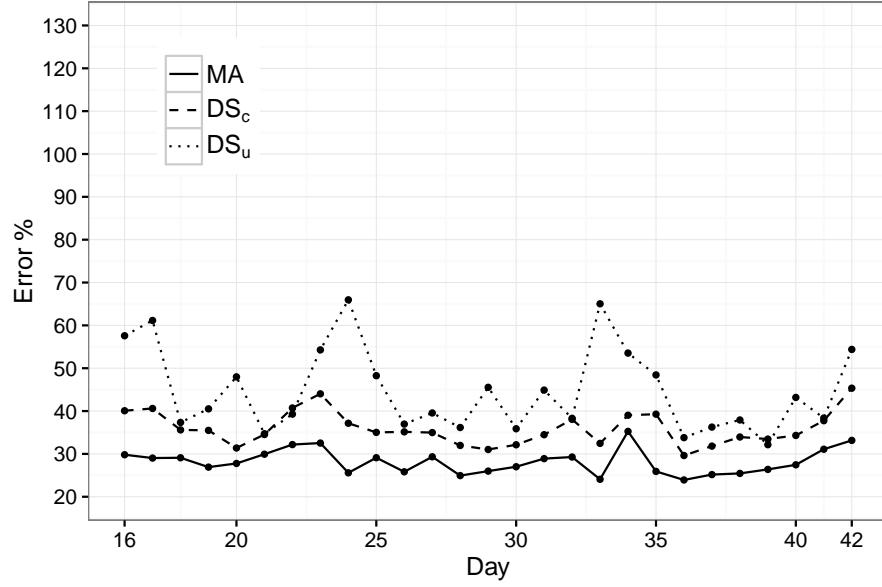


Fig. 3. Daily error curves (measured in percentages) for the one-step-ahead predictions with a sliding window of 20 days.

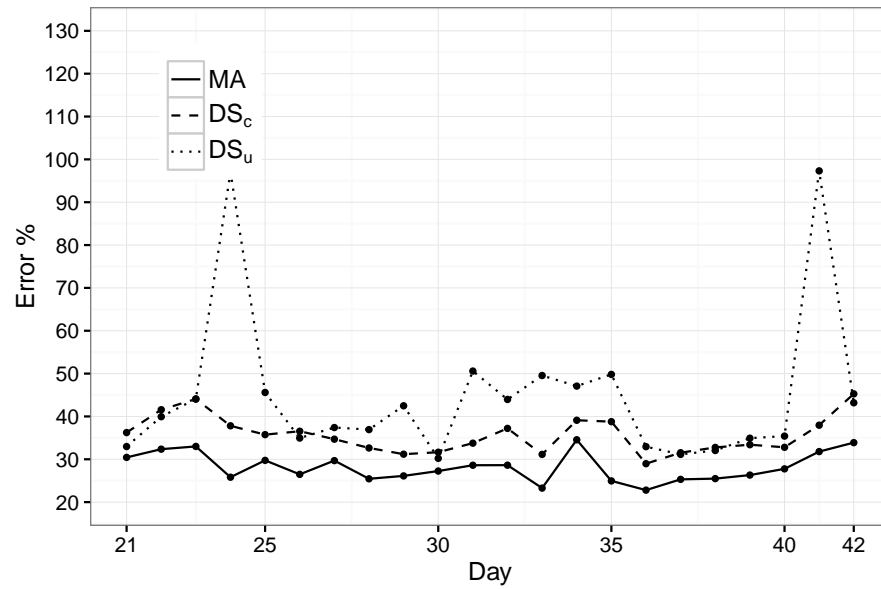
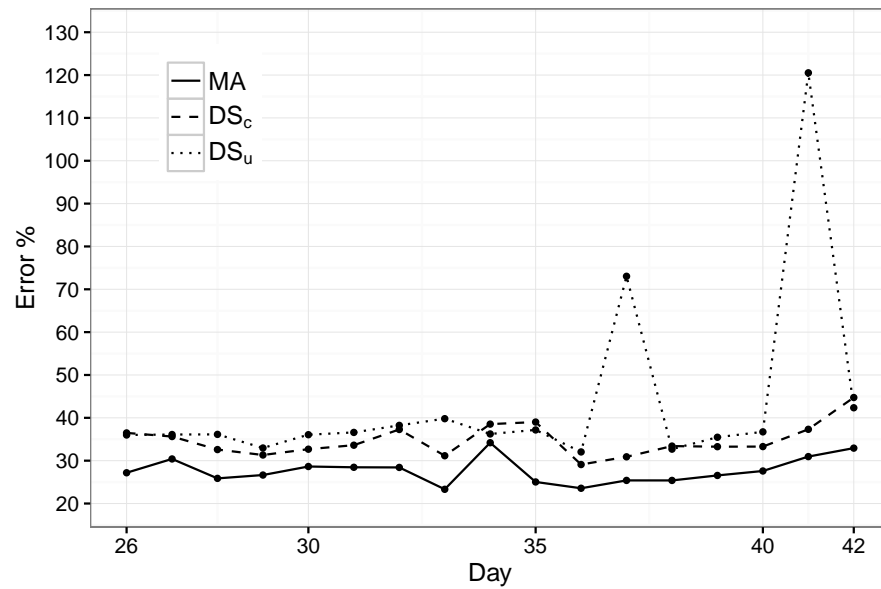


Fig. 4. Daily error curves (measured in percentages) for the one-step-ahead predictions with a sliding window of 25 days.



5 Conclusion

We evaluated three models to predict liquidity flows between banks in a payment system over time: a moving average model, a dynamic system, and a similar dynamic system with liquidity conservation constraint. The error of one-step-ahead predictions on real-world payment data revealed that the moving average model achieved the lowest error.

In the future we will conduct more evaluations of the models on larger data sets. In this paper we only used data of a small subset of banks in the TARGET2 payment system. The accumulated savings of the banks could not be exactly calculated in the dynamic systems. We will investigate if this negatively impacted the predictions. Moreover, we aim to extend the dynamic systems with the historic outflow of banks.

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